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Deflecting Modes in Proton Linacs (Aren't Important)

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A. Conditions for Resonance

Near the axis where the beam travels, the longitudinal component of the vector potential of a deflecting mode $T_{11(n)}$ with frequency ω_d in a gap region is approximately

$$A_z = B\, x \cos(\omega_d t+\phi) \cos(\pi n z/L) \qquad 1.1$$

where B is the magnetic field strength on the axis, ϕ is the phase which changes by ϕ_d between neighboring gap regions (cells), and n is the number of field sign alternations in a single gap of length L.

In order for a significant amount of energy to be exchanged between the beam and the deflecting mode, the phase of this term must remain constant as the beam moves through the field. Due to the accelerating RF field of frequency ω_a and the quadrupole focussing system, the beam has motions

$$x = \sqrt{(2\beta I)} \cos\sigma_x \quad \text{(betatron motion)}$$

$$t = z/v_0 + \Phi/\omega_a \cos\sigma_z \quad \text{(phase motion)} \qquad 1.2$$

where the phases σ_x and σ_z of betatron and phase motion change by μ_x and μ_z between neighboring gaps. Inserting these into 1.1, we find phases of the form

$$\pm\sigma_x \pm (\omega_d z/v + \phi \pm m\sigma_z) \pm n\pi z/L \qquad 1.3$$

To obtain resonance in passing through cells of length L (π mode), this phase must not change in one cell, to whit

$$\pm\mu_x \pm(\pi\omega_d/\omega_a + \phi_d \pm m\mu_z) \pm n\pi = 0 \qquad 1.4$$

We comment that terms of different m have relative strengths of $J_m(\omega_d\Phi/\omega_a)$. On a plot of ω vs ϕ_d, these conditions appear as lines of two different signs of slopes (see figure 4 p.30 of reference *1). If the curve ω_d vs ϕ_d for the mode intersects one of these lines, then resonant interchange of energy can take place, and the mode can grow from energy extracted from the beam. The most worrysome mode in the disc and washer (DAW) structure is apparently the TM_{110} whose frequency is close to the operating mode. The beam is tightly bunched in our case, $\Phi \approx \pi/12$, which might be taken as an upper limit on the amplitude of coherent oscillation. Then $J_m \approx (\pi/24)^m \ll 1$ for $m \neq 0$. Thus we need only consider the m = 0 motion. Inspection of figures #15 and B-2 to B-7 in reference #1 lead one to believe that this resonant condition is likely to be met in at least part of the 100-400 MeV linac section proposed for the Fermilab upgrade.

B. Strength of the Interaction

Considerable data exists for the DAW structure from cavities built for PIGMI and the NBS Race Track Microtron. This is summarized in reference #1. These cavities were at higher frequencies, about 2380 MHz, so some scaling of shunt impedances, Q's etc. should be done at some point. For now, we will use them directly. In ref. 1, p.33, The deflection for an eight meter cavity is calculated for the RTM. Presumably this was for the first pass through the cavity, in which case the final energy is 17 MeV. (Nowhere is this stated in the report.) The deflection is 1 mrad for a 1 Amp beam 1 mm off axis. The deflection angle is proportional to beam current, and inversely proportional to the relativistic mass $m\gamma$, Scaling by these factors (50 mA, 1000 MeV) we can define a quantity $k \approx 10^{-4} m^{-2}$ which is the deflection angle per meter of cavity per meter of offset of the beam for 100 MeV protons. We can make a worst case estimate of the deflection of the ion beam if we assume the beam has an offset of 2 mm in the 56 m of structure. The total deflection is about 10 µrad. With reasonable assumptions about the β function, the maximum offset is about 0.1 mm.

C. Landau Damping

The question remains as to whether the beam will actually oscillate or be damped by its frequency spread. The answer to this question is usually as follows. The fields induced by the beam cause a shift in the coherent oscillation frequency. If this shift is greater than the spread in this frequency in the beam, the beam will oscillate, and the instability will grow. Otherwise, the beam will not oscillate. The oscillation in question is the transverse (betatron) frequency. A reasonable choice for this is to choose a betatron phase advance of $\pi/3$ in one cell (2 m for two quads), or $\Omega \approx 0.5\ m^{-1}$. Using the smooth approximation for the coherent motion of the beam, including the deflecting mode with k defined above,

$$x'' + \Omega^2 x = -kx \qquad 1.5$$

$$\Delta\Omega = k/2\Omega \approx 2 \cdot 10^{-4}\ m^{-1} \qquad 1.6$$

The spread in the betatron frequency is due to the momentum spread $\Delta p/p$ which is about $2 \cdot 10^{-3}$

$$\delta\Omega = \Omega\ \Delta p/p \approx 10^{-3}\ m^{-1} \qquad 1.7$$

Thus it appears that the beam will be Landau damped and not oscillate at all. On the other hand it would be useful to check this result with more careful calculations, particularly by scaling the frequency, and developing the proper dispersion relation for the instability.

D. Transient Deflection

Since the beam will not oscillate, we can make a better estimate of the likely outcome. Each two meter RF cavity will be excited by the mean offset of the beam in it. While the mode is being established, the exit angle will change. The deflections of each cavity add up in their own (random) phase. We can estimate the r.m.s. of the expectation value of the deflection angle at the end of the linac. If we assume that the r.m.s. variation of beam position at each gap is 2 mm, we get the same result as in section B above, except it is divided by the square root of the total number of gaps (about 500). Then the expected angular variation at the end of the linac is about 1/2 µrad (σ). If on the other hand we assume that the r.m.s. variation corresponds to a complete cavity, the result is the same as in section B except it is divided by the square root of the number of cavities (about 25), so the σ of the expected variation is about 2.5 µrad. To do better than this one needs to develop a model including the actual betatron motion and correlation effects. Some real data from the 200 MHz section would be useful.

E. Further Comment

As stated above, the energy used in estimating the deflection in ref #1 was not stated. It is possible to use other results stated to infer its value. It turns out to be about 9 Mev, so the estimates above are a factor of two too pessimistic because of this.

The estimates referred to involve Formulas 4 and 5 of ref #1, which gives the deflection in terms of some constants and the mass and γ of the particle; the $\perp$ shunt impedance, Q, and transit time factor; and a "buildup factor" for the mode. The buildup factor is stated to be 10^3, but when I calculate it, I get 150. Subsequent to the publication of ref.#1, measurements were made at Argonne of the shunt impedances, frequencies and Q's of a variety of modes in a DAW structure at 1300 MHz (ref. #2). They report the shunt impedance term for TM_{110} ($LZ_{\perp}/Q$) to be about 5 kΩ/m^2 for a cavity with three washers terminated in half discs. The deflecting modes were not seen in "steady state" excitation, but only when a single short beam pulse was passed off axis through the cavity. In reference #1, the value used was that of a simple pillbox, 122 kΩ/m^2. In view of all this, the above estimates for the linac upgrade may be grossly pessimistic.

F. Conclusions

If there is economic or other advantage to using DAW structure for the upgrade, it is my belief that the deflecting modes, usually quoted as the downfall of DAW, should not be the reason to fail to exploit this advantage.

References:

1. R.K. Cooper et. al., **LA-UR**-83-95 (Los Alamos Internal Report,1983)
2. ....and G. S. Mavrogenes, IEEE Trans. Nuc. Sci., **NS-32**, #5, p. 2846, October 1985